Analog Computer Group Lab Report

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Introduction

Although rarely used in today's world, analog computers can be extremely useful in solving mathematical operations and differential equations. In this lab, an analog computer was designed and built using a combination of circuits to solve an ordinary differential equation modeling glucose transport and metabolism within a cell. The intake and consumption of glucose by a cell was simplified into one differential equation. The circuit was carefully designed in a way that the output voltage corresponded to the solution of the differential equation. The electrical analog solution was then compared with the analytical calculus solution as well as a computational MATLAB solution.

Methods and Materials

Glucose transport into red blood cells was modeled by the following differential equation, where X_0 represents blood glucose and S_1 represents intracellular glucose. V_1 represents the rate of glucose transport into and out of the cell, so $V_1 = K_1*X_0 - K_2*S_1$, and V_2 represents that rate of conversion of glucose to lactate, so $V_2 = K_3*S_1$.

$$\frac{dS_1}{dt} = V_1 - V_2 = K_1 * X_0 - K_2 * S_1 - K_3 * S_1$$

The analog computer was designed using a box, circuit, and wiring diagram to solve this differential equation and measure S_1 directly over time. K_1*X_0 was treated and applied to the circuit as a constant voltage and the value of S_1 at time zero was set to be zero.

The circuit was assembled on a breadboard according to the wiring diagram (Figures 4, 5, 6 in Appendix). All resistors except the one indicated R_5, which had a value of 1MΩ, had a value of 10kΩ. One 353 and two 741 operational amplifiers were used. One 1μF capacitor was used in the… One switch was implemented to allow the initial S_1 value to be set, and two potentiometers were used to change the values of K_2 and K_3 between settings (Table 1). The circuit was grounded, connected to the +/- 25V voltage supplies, and connected to the +6V voltage supply with the negative terminal grounded to provide the constant K_1*X_0. The output was recorded using an oscilloscope.

	K_1*X_0 Input Voltage (V)	Pot-1 R_1 (Ω)	Pot-1 R_2 (Ω)	Pot-2 R_1 (Ω)	Pot-2 R_2 (Ω)	K_2 Value (Ω)	K_3 Value (Ω)
Setting 1	1	4.963	4.866	4.965	4.825	0.495	0.493
Setting 2	1	2.203	7.634	4.239	5.541	0.776	0.567
Setting 3	2	2.203	7.634	4.239	5.541	0.776	0.567

Table 1. Input voltage, potentiometer (Pot) settings, and calculated constans K_2 and K_3 corresponding with each setting of the analog computer.

Results

The circuit was designed using a large assumption about X_0 and S_1. Since our model only allows for a heavily simplified version of the problem we were tasked with, X_0 is assumed to be a constant value for all of time and S_1 is initially set to be zero. In reality, X_0 is representative of the blood glucose level, which can vary dramatically, and the intracellular glucose concentration S_1 in human cells should never be 0. For our purposes, X_0 is not problematic to be left as a constant, but setting S_1 to be 0 is a fairly troublesome simplification. In trying to accurately model how glucose is transported over long time frames, both of the assumptions made would be an inappropriate.

The circuit also utilizes a switch to implement an initial condition for S_1 into our equation. Because the switch is grounded, the initial condition is applied as voltage of 0V, according to the simplification described above, and therefore was given a value of 0 in our computed solutions as well. Prior to the switch being turned on, no voltage is allowed into the integrator portion of our circuit, and an output signal of 0V can be seen. After flipping the switch, the circuit begins integrating and the solution curve is created on the oscilloscope. Flipping the switch back to ground discharges the capacitor, and enables us to repeat the initial observation.

After the switch allows voltage to travel to the integrator, it is important that appropriate values for R_5 and the capacitor are selected (Figure 5, Figure 6). This RC value relates to the ratio that the integrated voltage is multiplied by. Our RC value was intentionally selected so that it would be equal to 1, and the integrated voltage would then have a coefficient of 1. With an RC value not equal to 1, the voltage would have to be amplified by the final op-amp, which could introduce unwanted noise or gain in the final signal.

After the circuit was completed, it was run with the voltages and potentiometer values specified in Table 1. The results from each setting can be seen as the first image in Figures 1, 2, and 3 respectively. These settings were chosen to compare how changing the various variables in the equation would impact the output signal. The second image in each Figure below displays the analytical and computational solutions, as plotted in MATLAB. The analytical solution was found by hand, using a standard solving method. The computational solution was created by MATLAB using the ode45 function. The analog computer results were very similar to both the analytical and computational solution, which were nearly identical, for all three settings. Figure 1 shows the analog computer output voltage for setting one approaching slightly greater than 1V at at steady state, which is similar to the analytical and computational solutions that approach an intracellular glucose concentration of about 1 at steady state. Figures 2 and 3 show similar results, with the most noticeable difference between the analog computer and the analytical and computational solutions being the slight variation in the steady state reached. In all three Figures, the analog computer reached a higher steady state value than the computational solution plots. This could be due to error caused by the internal components, as the resistors, potentiometers, and a capacitor always have some error associated with them.
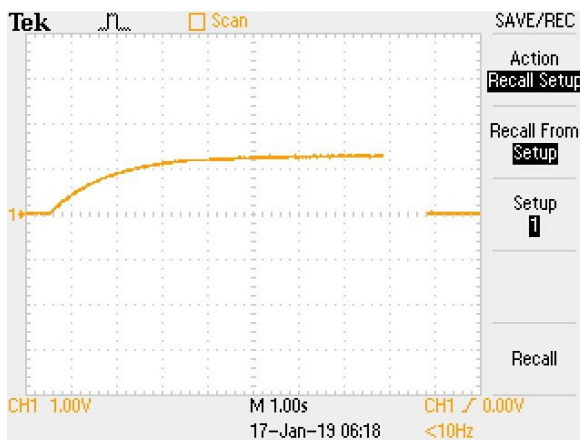

Figure 1. Setting one calculated by the analog computer, computationally, and analytically. Note the difference in the final steady state level. This can be seen in Figures 2 and 3 as well.
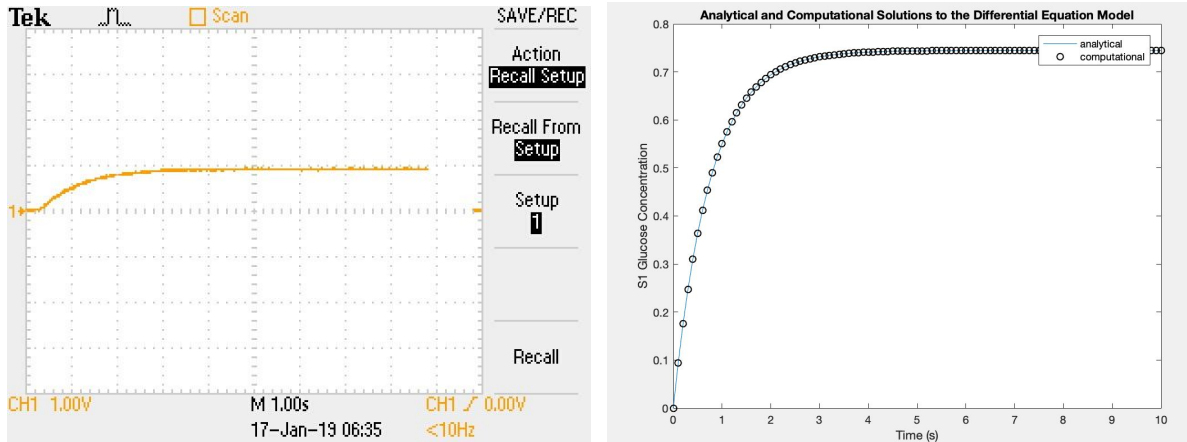
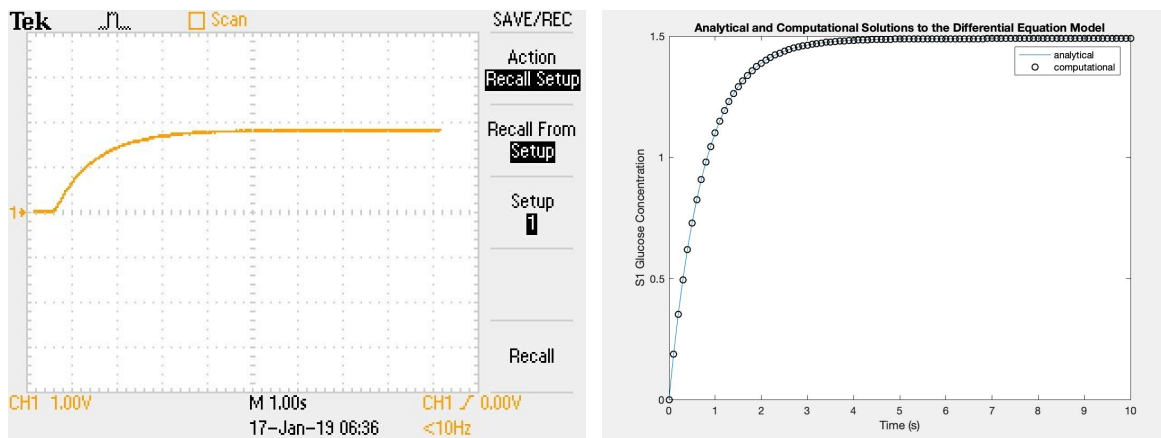**Figure 2.** Setting two calculated by the analog computer, computationally, and analytically.

Figure 3. Setting three calculated by the analog computer, computationally, and analytically.

Overall, our results make sense when considering the question that we originally set out to answer, and with the given simplifications. The differential equation listed above shows that over time, S_1 approaches $V_1 - V_2$. Considering the equation, if V_2 is greater than V_1, then the S_1 would appear similar to a damped, oscillating pattern. It would decrease, and then eventually approach 0, which would then make V_2 less than V_1, causing an increase again until equilibrium is reached. However, taking into consideration the system as a whole, V_2 cannot, at least initially, be greater than V_1, as S_1 is set to 0 initially. Because of this, we only see the steady increase of S_1 until steady state is reached, as there is no oscillation in our model. The analog computer generated plots and the analytical and computational solutions observed are consistent with this analysis of the given differential equation.

As briefly described above, X_0 represents blood glucose concentration in humans in our model. This level varies between individuals and fluctuates with food intake and physical activities. If no food is consumed for eight hours, the normal blood glucose concentration lies

between 70 to 99 mg/dL[1]. Within two hours of food consumption, the normal blood glucose concentration increases up to 140 mg/dL. For the model used in this lab, X_0 was assumed to be constant throughout and the intracellular glucose concentration (S_1) was set to zero initially. These assumptions were made to simplify the circuitry of the analog computer. If X_0 varied, then an extra circuit would be needed to vary the input. In addition, capacitors would needed to be pre-charged to emulate a non-zero initial condition. With that said, the X_0 assumption was reasonable as most of the time, the human blood glucose stays constant as energy is pulled out of storage to maintain a baseline blood glucose concentration. The S_1 assumption was less reasonable as the intracellular glucose concentration never reaches zero in a human body. However, the assumption was needed to create a simple enough model that can be built in the scope of this lab.

Conclusions

This lab was very insightful in learning about analog computer circuits and solving differential equations. Specifically, the group learned to model a biological process with a simple differential equation. In addition, the group was introduced to illustrating various circuit diagrams and also learned how different op-amps and potentiometers work. These skills are extremely applicable since bioengineers are often tasked with using models, circuitry and mathematics to solve biological problems in team settings. Furthermore, this lab allowed the group to apply the knowledge learned in the Systems and Control course regarding feedback in the context of circuitry and differential equations. Although this exercise simplified a real biological process, the critical thinking and technical skills acquired by the group set a solid foundation for future applications in bioengineering.

Appendix

References

1. "What Are Normal Blood Glucose Levels?" Virginia Mason Hospital & Medical Center, www.virginiamason.org/WhatareNormalBloodGlucoseLevels.
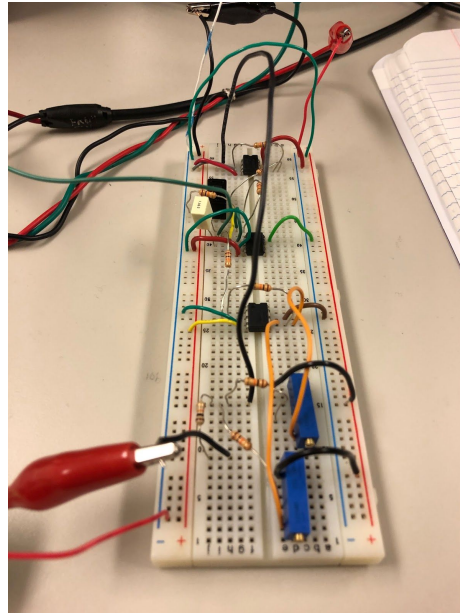
Circuit Figures

Figure 4. Final analog computer circuit implementation. A more clear depiction of the circuit can be seen in Figures 5 and 6.

Figure 5. Final group circuit diagram used for assembly. Resistors and capacitor are labeled (R_x or C) as well as the voltage coming from various paths (e_x). These voltages were used to calculate the proper values for the resistors and capacitor.

Figure 6. Final wiring diagram used for assembly. Values for various resistors and capacitors are included. V_0 and S_0 label the points to input voltage and measure voltage, respectively.

Figure 7. Analytical solution hand-written calculations solving for S_1 as a function of time.

MATLAB Code

```
clear all; close all; clc;
%Define variables
k1 = 1;
%k2 = 0.495;
k2 = 0.776;
%k3 = 0.493;
k3 = 0.567;
%X0 = 1;
X0 = 2;

%Analytical Solution
tspan = 0:0.1:10;
S1anal = (k1*X0/(k2+k3))*(1-exp(-(k2+k3).*tspan));

plot(tspan,S1anal)
hold on

%Computational Solution
y0 = 0;
[t,S1comp] = ode45(@fun1, tspan, y0);
plot(t,S1comp,'ko')
legend('analytical','computational')
title('Analytical and Computational Solutions to the Differential Equation Model')
xlabel('Time (s)')
ylabel('S1 Glucose Concentration')
hold off

function dS1dt = fun1(t,S1)
%function that models the differential equation
k1 = 1;
%k2 = 0.495;
k2 = 0.776;
%k3 = 0.493;
k3 = 0.567;
%X0 = 1;
X0 = 2;
dS1dt = k1*X0 - k2*S1 - k3*S1;
end
```